Analysis of Radial Velocity Measurements
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1. Introduction
The existence of solar systems other than our own has been speculated for a long time, but actually finding exoplanets around other stars is only possible since very recently: the first published discovery to receive subsequent confirmation was made in 1988 (Campbell et al. [1988], Hatzes et al. [2003]). Since then, a multitude of planets was discovered using various methods. One of those methods, the most productive so far, is the Radial Velocity (RV) method (also known as Doppler spectroscopy).

In this work I analyze two different time series of RV measurements corresponding to two stars which have orbiting planets. I will first consider a system where a circular orbit is a good approximation, and try to fit a sinusoidal function to the data, in order to derive physical parameters of the planet. The general signal produced by an elliptic orbit is then studied and a Keplerian function is fitted to RV measurements of a second star. The planets' mass and semi-major axis are calculated from the fitted parameters.

2. Theoretical aspects of the method
The radial velocity method does not detect the planet directly. Instead, it uses the gravitational effect of the planet on the star. Both star and planet orbit the common system barycenter (in this case, the center of mass) in elliptical orbits. This means that the star will undergo a periodic change in (projected in the line of sight, or radial) velocity. By using Doppler blueshifts and redshifts in the star’s spectrum, we can measure this radial velocity change and infer the presence of an orbiting planet.

The elliptical orbit of the star is characterized (like all ellipses) by the semi-major axis $a$ and the eccentricity $e$ that give the shape of the ellipse, and by three angles that determine its spatial orientation in the sky (Figure 1). Within the orbital plane, three more angles are used to describe the position of the (center of the) star at a particular time: the true, eccentric and mean anomalies. The true anomaly $\nu(t)$ is the angle between the direction of pericentre and the current position of the body measured from the barycentric focus of the ellipse. It is not linear with time due to Kepler’s second law. The eccentric anomaly $E(t)$ is related to the true anomaly by the following formula.
\[ \tan \frac{\nu(t)}{2} = \left( \frac{1 + e}{1 - e} \right)^{1/2} \tan \frac{E(t)}{2} \]

The mean anomaly \( M(t) \) is given by

\[ M(t) = \frac{2\pi}{P} (t - \tau) \]

where \( P \) is the orbital period of the star (equal to the orbital period of the planet) and \( \tau \) is the time of pericentre passage. The following relation, Kepler’s equation, also holds

\[ M(t) = E(t) - e \sin E(t) \]

If we consider Figure (1) as representing the star’s orbit around the barycenter, it is easy to derive an expression to the \( z \) coordinate along the line of sight (Clubb [2008]):

\[ z(t) = r(t) \sin i \sin(\omega + \nu(t)) \]

The radial velocity of the star is then given by

\[ V_r(t) \equiv \dot{z} = K \left[ \cos(\omega + \nu) + e \cos \omega \right] + \gamma \]

where \( K \) is the semi-amplitude of the signal (defined below) and \( \gamma \) the average radial velocity of the system.

3. Approximating a circular orbit

In this first exercise we analyzed radial velocity measurements of a star that has a giant planet in orbit. The provided data spanned approximately 150 Julian days (with times not evenly spaced) and included the measurement errors. Figure (2) shows a plot of the time series. Assuming the planet moves in a circular orbit, we tried to fit to the data a function of the type (5). We provided initial guesses to the parameters that seemed appropriate:

- \( K_{\text{guess}} = 0.1 \)
- \( P_{\text{guess}} = 25 \)
- \( \gamma_{\text{guess}} = -10.8 \)

The fitted line is also shown in Figure (2), together with the residuals of the fit. The best-fit parameters are

- \( P_{\text{fit}} = 24.33 \) days
- \( \tau_{\text{fit}} = 5.11 \times 10^4 \) days
- \( K_{\text{fit}} = 0.0617 \) km/s
- \( \gamma_{\text{fit}} = -10.816 \) km/s

From the fitted parameters it is possible to derive (a minimum value for) the mass of the planet and the semi-major axis of the orbit. We use the following equation (which also defines the semi-amplitude \( K \))

\[ \frac{M_p^3 \sin^3 i}{M_p + M_*} = 1.036 \times 10^{-7} K^3 P (1 - e^2)^{3/2} \]

where \( M_* \) is the mass of the star, \( M_p \) is the mass of the planet (both in solar masses) and the eccentricity \( e \) is obviously zero in this case. The result is also in solar masses. We assume a star with \( 1M_\odot \) and \( M_p \ll M_* \). The resulting planet mass is

\[ M_p \sin i = 8.3946 \times 10^{-4} \ M_\odot = 0.8794 \ M_{\text{Jup}} \]

Kepler’s third law can be used to derive the semi-major axis of the orbit:

\[ P^2 = \frac{a^3}{M_p + M_*} \]

where both masses are in solar mass units, the period \( P \) in years and \( a \) is obtained in astronomical units (AU). The result obtained was

\[ a = 0.1644 \text{ AU} = 2.459 \times 10^{10} \text{ m} \]

We see that this is a giant planet with almost 90% the mass of Jupiter, orbiting the star at a distance around half of that of Mercury from the Sun. This reflects the current observational bias of the RV technique: it is easier to detect massive planets with short periods for these will create the most discernible effect on the star.
4. The shape of the Keplerian function

In the general case of an elliptic orbit, the presence of a planet induces a movement in the star that results in a signal of the form (4). It is important to note that the radial velocity of the star does not depend explicitly on time, but rather on the true anomaly. This angle, \( \nu(t) \), can be calculated from equations (1), (2) and (3). In the case of eq. (3), which is transcendental, we used a method called Lagrange Expansion to calculate the eccentric anomaly \( E \) (see Moulton [1984] for a full discussion).

The shape of the radial velocity curve \( V_r(t) \) is determined by \( e \) and \( w \). Shown in Figure (3) are four plots of the Keplerian function, considering four different sets of parameters (with changes in \( e \) and \( w \) only):

a) \( P=100 \text{ days} \quad e = 0.3 \quad K=0.1 \text{ km/s} \quad \gamma = 0 \quad T=0 \quad \omega = 0 \)
b) \( P=100 \text{ days} \quad e = 0.3 \quad K=0.1 \text{ km/s} \quad \gamma = 0 \quad T=0 \quad \omega = \pi \)
c) \( P=100 \text{ days} \quad e = 0.6 \quad K=0.1 \text{ km/s} \quad \gamma = 0 \quad T=0 \quad \omega = 0 \)
d) \( P=100 \text{ days} \quad e = 0.0 \quad K=0.1 \text{ km/s} \quad \gamma = 0 \quad T=0 \quad \omega = 0 \)

5. Fitting a Keplerian function to the radial velocity data

In Section 3, it was clear from Figure (2) that the time series had some kind of periodic variation. The data from the second star we studied is not so “well behaved” (Figure 5). In this case, a planet moving on a circular orbit is not a good approximation. A signal of the type (4) is then expected to represent the variation of the radial velocity, and the question is to find the parameters \( P, e, K, \gamma, \tau \) and \( \omega \) that best fit the points. The non-linearity of the parameters describing a radial velocity curve makes this a non-trivial task. A minimization algorithm, that fits the data in the least-squares sense, was used.

Let \( t \) be the vector of time instants where there is a measured RV. In each step, the algorithm has to calculate the true anomaly corresponding to the set of parameters \( (P, e, \tau) \), at all instants in \( t \). Given \( \nu(t) \), a Keplerian test function can be calculated and the residuals \( (V_r^{\text{test}}(t) - V_r(t)) \) evaluated. The algorithm thus searches the six-dimensional \( \chi^2 \) space trying to find the global minimum of the residual function, i.e. the parameters that best fit the data points.

We refer in this work to a MATLAB implementation of nonlinear least squares fitting, \texttt{LSQCURVEFIT}. This routine allows for the use of two different optimization algorithms: Trust-Region-Reflective and Levenberg-Marquardt (Levenberg
Figure 3: Four examples of radial velocity curves, showing the dependence on $e$ and $\omega$. The dashed line represents the systemic velocity.

[1944, Marquardt [1963], Press et al. [1992]). Using the first algorithm it is possible to define lower and upper bounds to the parameters. Given good initial estimates of the parameters, both methods converge to the same solution with no noticeable difference in speed.

To provide a good estimate of the period of the orbiting planet, we constructed a Lomb-Scargle periodogram (Lomb [1976], Scargle [1982]) of the data and identified the frequency associated with the tallest peak (Figure 4). The remaining initial guesses were set by simple inspection of the data.

In Figure (5) we plot the RV measurements together with the fitted Keplerian function. The fitted parameters are

\[ P_{fit} = 776.02 \text{ days} \]
\[ e_{fit} = 0.42 \]
\[ K_{fit} = 0.04 \text{ km/s} \]
\[ \gamma_{fit} = 39.31 \text{ km/s} \]
\[ \tau_{fit} = 53026 \text{ days} \]
\[ \omega_{fit} = 5.71 \]

The residuals corresponding to this fit are shown in the bottom panel. From the scatter of the residuals (the absence of periodical structure) we can infer that there is no periodical signal in the RV data, other than the fitted planet. A measure of the goodness of fit is given by the root mean square of the residuals. We get a value of 0.0047 in this case, suggesting a fairly good fit to the data.

Just as in Section 3, it is possible to derive the mass and semi-major axis of the orbiting planet. Using expressions (6) and (7) and again considering that the star has $1M_\odot$, the derived values are

\[ M_p = 1.677 \ M_{Jup} \]
\[ a = 1.654 \ AU \]

which means that this second planet is also a giant but orbiting at a bigger distance from its star (approximately the same distance as Mars orbits the Sun).
Figure 4: Lomb-Scargle periodogram applied to the RV observations of the second star. The highest peak corresponds to $P = 775 \text{ d}$ and the best-fit solution gives $P = 776 \text{ d}$. The dashed lines show the significance levels of the peaks (a smaller value indicates a highly significant periodic signal).

Figure 5: Radial-velocity measurements of the second star as a function of time. The solid line represents the best Keplerian fit to the data: $P = 776 \text{ d}$; $e = 0.42$. The residuals of the fit are shown in the lower panel.
6. Conclusions

The radial velocity method has led to the discovery of over five hundred planetary systems. The increase in precision is expanding the parameter space to longer periods and smaller masses. The effect of a single planet in orbit of a star is well understood and modeled, but with increasing numbers of multi-planetary systems, planet-planet interactions must be considered, hardening the analysis.

In this work I studied the effect of an orbiting planet on its host star. Two cases, one where a circular orbit is a good approximation and one where it isn’t, were considered and in both general parametric curves were fitted to the RV observations. The results were two very different planets: the first, a typical, Jupiter-like giant, orbiting very near its host star, in a short period; and the second, a massive planet whose orbit extends past 1 AU. This two examples already show what the radial velocity method brought to our understanding of planet formation: a clear sign that our solar system is just a special case of a very large set of possibilities who need yet theoretical explanation.

References


